Rank Quasi-Cyclic (RQC)

Second round version

RQC is an IND-CCA2 KEM running for standardization to NIST's competition in the category "post-quantum public key encryption scheme". Different sets of parameters are proposed for security strength categories 1, 3, and 5.

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Contents

1 Specifications

In this section, we introduce RQC, an efficient encryption scheme based on coding theory. RQC stands for Rank Quasi-Cyclic. This proposal has been published in IEEE Transactions on Information Theory [1]. Many notations, definitions and properties are very similar to [7]. We nevertheless include them in this proposal for completeness.

RQC is a code-based public key cryptosystem with several desirable properties:

- It is proved IND-CPA assuming the hardness of (a decisional version of) the Syndrome Decoding problem on structured codes. By construction, RQC perfectly fits the recent KEM-DEM transformation of [17], and allows to get an hybrid encryption scheme with strong security guarantees (IND-CCA2) and good efficiency.
- In contrast with most code-based cryptosystems, the assumption that the family of codes being used is indistinguishable among random codes is no longer required.
- The decryption algorithm is deterministic so the Decryption Failure Rate (DFR) is zero.
- It features more attractive parameters than most of the Hamming based proposals.

Organization of the Specifications. This section is organized as follows: we provide the required background in Sec. 1.1, we make some recalls on encryption and security in Sec. 1.1.5 then present our proposal in Sec. 1.2. Concrete sets of parameters are provided in Sec. 1.4.

1.1 Preliminaries

1.1.1 General definitions

In the following document, q denotes a power of a prime p. The finite field with q elements is denoted by $\mathbb{F}_q$ and more generally for any positive integer m the finite field with q^m elements is denoted by $\mathbb{F}_{q^m}$. We will frequently view $\mathbb{F}_{q^m}$ as an m-dimensional vector space over $\mathbb{F}_q$.

We use bold lowercase (resp. uppercase) letters to denote vectors (resp. matrices).

Let $P \in \mathbb{F}_q[X]$ a polynomial of degree n. We can identify the vector space $\mathbb{F}_{q^m}^n$ with the ring $\mathbb{F}_{q^m}[X]/\langle P\rangle$, where $\langle P\rangle$ denotes the ideal of $\mathbb{F}_{q^m}[X]$ generated by P.

$$\begin{aligned} \Psi : \quad \mathbb{F}_{q^m}^n &\simeq \mathbb{F}_{q^m}[X]/\langle P\rangle \\ (v_0, \ldots, v_{n-1}) &\mapsto \sum_{i=0}^{n-1} v_i X^i \end{aligned}$$

For $\mathbf{u}, \mathbf{v} \in \mathbb{F}_{q^m}^n$, we define their product similarly as in $\mathbb{F}_{q^m}[X]/\langle P\rangle$: $\mathbf{w} = \mathbf{uv} \in \mathbb{F}_{q^m}^n$ is the only vector such that $\Psi(\mathbf{w}) = \Psi(\mathbf{u})\Psi(\mathbf{v})$. In order to lighten the formula, we will omit the symbol Ψ in the future.

To a vector $\mathbf{v} \in \mathbb{F}_{q^m}^n$ we can associate an $n \times n$ square matrix with entries in $\mathbb{F}_{q^m}^n$ corresponding to the product by $\mathbf{v}$. Indeed,

$$\begin{aligned}
\mathbf{u} \cdot \mathbf{v} &= \mathbf{u}(X)\mathbf{v}(X) \pmod P \\
&= \sum_{i=0}^{n-1} u_i X^i \mathbf{v}(X) \pmod P \\
&= \sum_{i=0}^{n-1} u_i (X^i \mathbf{v}(X) \mod P) \\
&= (u_0, \ldots, u_{n-1}) \begin{pmatrix} \mathbf{v}(X) & \mod P \\ X\mathbf{v}(X) & \mod P \\ & \vdots \\ X^{n-1}\mathbf{v}(X) & \mod P \end{pmatrix}
\end{aligned}$$

Such a matrix is called the ideal matrix generated by $\mathbf{v}$ and P, or simply by $\mathbf{v}$ when there is no ambiguity in the choice of P.

Definition 1.1.1 (Ideal Matrix)**.** *Let $P \in \mathbb{F}_q[X]$ a polynomial of degree n and $\mathbf{v} \in \mathbb{F}_{q^m}^n$. The ideal matrix generated by $\mathbf{v}$ is the $n \times n$ square matrix denoted $\mathcal{IM}(\mathbf{v})$ of the form:*

$$\mathcal{IM}(\mathbf{v}) = \begin{pmatrix} \mathbf{v} & \\ X\mathbf{v} & \mod P \\ & \vdots \\ X^{n-1}\mathbf{v} & \mod P \end{pmatrix}$$

As a consequence, the product of two elements of $\mathbb{F}_{q^m}[X]/\langle P \rangle$ is equivalent to the usual vector-matrix product:

$$\mathbf{u} \cdot \mathbf{v} = \mathbf{u}\mathcal{IM}(\mathbf{v}) = \mathcal{IM}(\mathbf{u})^T \mathbf{v} = \mathbf{v} \cdot \mathbf{u}.$$

Definition 1.1.2 (Rank metric over $\mathbb{F}_{q^m}^n$)**.** *Let $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{F}_{q^m}^n$ and $(\beta_1, \ldots, \beta_m) \in \mathbb{F}_{q^m}^m$ a basis of $\mathbb{F}_{q^m}$ viewed as an m-dimensional vector space over $\mathbb{F}_q$. Each coordinate x_j is associated to a vector of $\mathbb{F}_q^m$ in this basis: $x_j = \sum_{i=1}^m x_{ij}\beta_i$. The $m \times n$ matrix associated to $\mathbf{x}$ is given by $\mathbf{M}(\mathbf{x}) = (x_{ij})_{\substack{1 \leqslant i \leqslant m \\ 1 \leqslant j \leqslant n}}$.*

The rank weight $\|\mathbf{x}\|$ of $\mathbf{x}$ is defined as

$$\|\mathbf{x}\| \stackrel{def}{=} \operatorname{Rank} \mathbf{M}(\mathbf{x}).$$

The associated distance $d(\mathbf{x}, \mathbf{y})$ between elements $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{F}_{q^m}^n$ is defined by $d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|$.

Definition 1.1.3 ($\mathbb{F}_{q^m}$-linear code)**.** *An $\mathbb{F}_{q^m}$-linear code $\mathcal{C}$ of dimension k and length n is a subspace of dimension k of $\mathbb{F}_{q^m}^n$ embedded with the rank metric. It is denoted $[n, k]_{q^m}$.*

Such a code $\mathcal{C}$ can be represented by two equivalent ways:

- *by a generator matrix* $\mathbf{G} \in \mathbb{F}_{q^m}^{k \times n}$. *Each row of* $\mathbf{G}$ *is an element of a basis of* $\mathcal{C}$,

$$\mathcal{C} = \{\mathbf{x}\mathbf{G}, \mathbf{x} \in \mathbb{F}_{q^m}^k\}.$$

- *by a parity-check matrix* $\mathbf{H} \in \mathbb{F}_{q^m}^{(n-k) \times n}$. *Each row of* $\mathbf{H}$ *determines a parity-check equation verified by the elements of* $\mathcal{C}$*:*

$$\mathcal{C} = \{\mathbf{x} \in \mathbb{F}_{q^m}^n : \mathbf{H}\mathbf{x}^T = \mathbf{0}\}.$$

  $\mathbf{H}\mathbf{v}^T$ *is called the* syndrome *of* $\mathbf{v}$ *(with respect to* $\mathbf{H}$*).*

We say that $\mathbf{G}$ *(respectively* $\mathbf{H}$*) is under systematic form if and only if it is of the form* $(\mathbf{I}_k|\mathbf{A})$ *(respectively* $(\mathbf{I}_{n-k}|\mathbf{B})$*).*

Definition 1.1.4 (Support of a word)**.** *Let* $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{F}_{q^m}^n$. *The support* E *of* $\mathbf{x}$*, denoted* $\mathrm{Supp}(\mathbf{x})$*, is the* $\mathbb{F}_q$*-subspace of* $\mathbb{F}_{q^m}$ *generated by the coordinates of* $\mathbf{x}$*:*

$$E = \langle x_1, \ldots, x_n \rangle_{\mathbb{F}_q}$$

and we have $\dim E = \|\mathbf{x}\|$.

The number of supports of dimension w of $\mathbb{F}_{q^m}$ is denoted by the Gaussian coefficient

$$\begin{bmatrix} m \\ w \end{bmatrix}_q = \prod_{i=0}^{w-1} \frac{q^m - q^i}{q^w - q^i}.$$

1.1.2 Ideal codes

One of the difficulty with code-based cryptography is the size of the key. Indeed, to represent an $[n,k]_{q^m}$ code with a systematic matrix, we need $k(n-k)$ symbols in $\mathbb{F}_{q^m}$, or $k(n-k)m\lceil \log q \rceil$ bits. In order to reduce the size of the representation of a code, we introduce the family of ideal codes, which are basically codes with a systematic generator matrix formed with blocks of ideal matrices. More formally,

Definition 1.1.5 (Ideal codes)**.** *Let* $P(X) \in \mathbb{F}_q[X]$ *be a polynomial of degree* n. *An* $[ns, nt]_{q^m}$ *code* $\mathcal{C}$ *is an* (s,t)*-ideal code if its generator matrix under systematic form is of the form*

$$\mathbf{G} = \begin{pmatrix} & \mathcal{IM}(\mathbf{g}_{1,1}) & \ldots & \mathcal{IM}(\mathbf{g}_{1,s-t}) \\ \mathbf{I}_{tn} & \vdots & \ddots & \vdots \\ & \mathcal{IM}(\mathbf{g}_{t,1}) & \ldots & \mathcal{IM}(\mathbf{g}_{t,s-t}) \end{pmatrix}$$

where $(\mathbf{g}_{i,j})_{\substack{i \in [1..s-t] \\ j \in [1..t]}}$ *are vectors of* $\mathbb{F}_{q^m}^n$. *In this case, we say that* $\mathcal{C}$ *is generated by the* $(\mathbf{g}_{i,j})$.

It would be somewhat more natural to choose the generator matrix to be made up of $s \times t$ ideal matrices, rather than to require the code to admit a systematic generator matrix. However, if m and n are two different prime numbers and if P is irreducible, a non-zero ideal matrix is always non-singular. To prove this, we need the following lemma:

Lemma 1. *Let m and n be two different prime numbers. Let $P \in \mathbb{F}_q[X]$ be an irreducible polynomial of degree n and $U \in \mathbb{F}_{q^m}[X]$ a non zero polynomial of degree at most $n-1$. Then P and U are co-prime in $\mathbb{F}_{q^m}[X]$.*

Proof. We will show that P and U have no common root. Let $\mathcal{Z}(P)$ (respectively $\mathcal{Z}(U)$) be the set of the roots of P (respectively U) in an algebraic closure of $\mathbb{F}_q$.

Since P is irreducible of degree n, its roots generate $\mathbb{F}_{q^n}$

$$\implies \mathcal{Z}(P) \subset \mathbb{F}_{q^n} \backslash \mathbb{F}_q$$

Since U is of degree at most $n-1$, its roots belong to $\mathbb{F}_{q^{m(n-1)!}}$.

But $\text{GCD}(n, m(n-1)!) = 1$ when m and n are two different prime numbers. Thus

$$\mathbb{F}_{q^{m(n-1)!}} \cap \mathbb{F}_{q^n} = \mathbb{F}_q \implies \mathcal{Z}(P) \cap \mathcal{Z}(U) = \emptyset$$

Hence, P and U are co-prime. □

Now, let $\mathbf{u} \in \mathbb{F}_{q^m}^n$ a non zero vector and $P \in \mathbb{F}_q[X]$ an irreducible polynomial of degree n. According to the previous lemma, there exists a vector $\mathbf{v} \in \mathbb{F}_{q^m}^n$ such that

$$\begin{aligned} & \mathbf{uv} = 1 \pmod{P} \\ \iff & \mathbf{u}\mathcal{IM}(\mathbf{v}) = (1, 0, \ldots, 0) \\ \iff & \mathcal{IM}(\mathbf{u})\mathcal{IM}(\mathbf{v}) = \mathbf{I}_n \end{aligned}$$

This demonstrates that every block of ideal matrix of $\mathbf{G}$ is non-singular, hence $\mathcal{C}$ can be represented under systematic form. □

All the parameters we propose in Section 1.4 verify these conditions.

Remark 1.1. *With this definition, ideal codes can be seen as a generalization of Quasi-Cyclic codes. Indeed, the generator matrix under systematic form of a Quasi-Cyclic code [10] is of the same form, except that the ideal matrices are replaced by circulant matrices. Yet, an $n \times n$ circulant matrix can be seen as an element of $\mathbb{F}_{q^m}[X]/\langle X^n - 1 \rangle$. Thus ideal codes only differ from Quasi-Cyclic codes by the choice of the polynomial P.*

In our scheme, we only use $[ns, n]_{q^m}$ ideal codes. In order to shorten the notation, we denote these codes an s-ideal code. If $\mathcal{C}$ is an $[sn, n]$ ideal code generated by $(\mathbf{g}_1, \ldots, \mathbf{g}_{s-1})$, we have $\mathcal{C} = \{(\mathbf{u}, \mathbf{ug}_1, \ldots, \mathbf{ug}_{s-1}), \mathbf{u} \in \mathbb{F}_{q^m}^n\}$.

We need to be careful when we use these notations in the case of parity-check matrix. Indeed, the parity-check matrix under systematic form of $\mathcal{C}$ is of the form:

$$\mathbf{H} = \begin{pmatrix} & \mathcal{IM}(\mathbf{h}_1)^T \\ \mathbf{I}_{n(s-1)} & \vdots \\ & \mathcal{IM}(\mathbf{h}_{s-1})^T \end{pmatrix}. \tag{1}$$

Thus, if $\boldsymbol{\sigma} = (\boldsymbol{\sigma}_1 \dots \boldsymbol{\sigma}_{s-1}) \in \mathbb{F}_{q^m}^{s(n-1)}$ is the syndrome of an error $\mathbf{e} = (\mathbf{e}_1 \dots \mathbf{e}_{s-1}) \in \mathbb{F}_{q^m}^{ns}$, the parity-check equations

$$\mathbf{H}\mathbf{e}^T = \boldsymbol{\sigma}^T$$

are equivalent to $\mathbf{e}_i + \mathbf{h}_i \mathbf{e}_{s-1} = \boldsymbol{\sigma}_i$ for $1 \leqslant i \leqslant s-1$.

Bounds for rank metric codes. The classical bounds for Hamming metric have straightforward rank metric analogues.

Singleton Bound. The classical Singleton bound for linear $[n, k]$ codes of minimum rank r over $\mathbb{F}_{q^m}$ applies naturally in the rank metric setting. It works in the same way as for linear codes (by finding an information set) and reads $r \leq 1 + n - k$. When $n > m$ this bound can be rewritten [19] as

$$r \leq 1 + \left\lfloor \frac{(n-k)m}{n} \right\rfloor . \tag{2}$$

Codes achieving this bound are called Maximum Rank Distance codes (MRD).

Deterministic Decoding. Unlike the situation for the Hamming metric, there do not exist many families of codes for the rank metric which are able to decode rank errors efficiently up to a given weight. When we are dealing with deterministic decoding, there is essentially only one known family of rank codes which can decode efficiently: the family of Gabidulin codes [8]. More details about these codes are provided in the next subsection. In a nutshell, they are defined over $\mathbb{F}_{q^m}$ and for $k \leq n \leq m$, Gabidulin codes of length n and dimension k are optimal and satisfy the Singleton bound for $m = n$ with minimum distance $d = n - k + 1$. They can decode up to $\lfloor \frac{n-k}{2} \rfloor$ rank errors in a deterministic way.

Probabilistic Decoding. There also exists a simple family of codes which has been described for the subspace metric in [25] and can be straightforwardly adapted to the rank metric. These codes reach asymptotically the equivalent of the Gilbert-Varshamov bound for the rank metric, however their non-zero probability of decoding failure makes them less interesting for the cases we consider in this paper.

1.1.3 Gabidulin codes and their decoding

Gabidulin codes were introduced in 1985 [8]. These codes are analog to Reed-Solomon codes in Hamming metric [23], but involve q-polynomials instead of regular ones. They have therefore a strong algebraic structure. The notion of q-polynomials was introduced by Ore [21], we hereafter give some background.

Definition 1.1.6 (q-polynomials)**.** *The set of q-polynomials over $\mathbb{F}_{q^m}$ is the set of polynomials with the following shape:*

$$\left\{ P(X) = \sum_{i=0}^{r} p_i X^{q^i}, \ \textit{with } p_i \in \mathbb{F}_{q^m} \ \textit{and } p_r \neq 0 \right\}.$$

The q-degree of a q-polynomial P is defined as $\deg_q(P) = r$.

Definition 1.1.7 (Ring of q-polynomials)**.** *The set of q-polynomials over $\mathbb{F}_{q^m}$ is a non-commutative ring when considered with the following operations:*

- *Addition:* $(P+Q)(X) = P(X) + Q(X)$
- *Composition:* $(P \circ Q)(X) = P[Q(X)]$

Due to their structure, the q-polynomials are inherently related to decoding problems in the rank metric as stated by the following propositions.

Theorem 1.2 ([21])**.** *Any $\mathbb{F}_q$-subspace of $\mathbb{F}_{q^m}$ of dimension r is the set of the roots of a unique unitary q-polynomial P such that* $\deg_q(P) = r$.

Corollary 1.1. *Let $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \mathbb{F}_{q^m}^n$ and V be the unitary q-polynomial of smallest q-degree such that $V(x_i) = 0$ for $1 \leq i \leq n$, then $\|\mathbf{x}\| = r$ if and only if* $\deg_q(V) = r$.

Gabidulin codes can be thought as the evaluation of q-polynomials of bounded degree on the coordinates of a vector over $\mathbb{F}_{q^m}$.

Definition 1.1.8 (Gabidulin codes)**.** *Let $k, n, m \in \mathbb{N}$ such that $k \leqslant n \leqslant m$. Let $\mathbf{g} = (g_1, \ldots, g_n)$ be a $\mathbb{F}_q$ linearly independent family of elements of $\mathbb{F}_{q^m}$. The* Gabidulin code $\mathcal{G}_{\mathbf{g}}(n, k, m)$ *is the following code $[n, k]_{q^m}$:*

$$\left\{P(\mathbf{g}), \deg_q P < k\right\} \text{ where } P(\mathbf{g}) := (P(g_1), \ldots, P(g_v)).$$

A generator matrix for $\mathcal{G}_{\mathbf{g}}$ is given by:

$$\mathbf{G} = \begin{pmatrix} g_1 & \cdots & g_n \\ g_1^q & \cdots & g_n^q \\ \vdots & \ddots & \vdots \\ g_1^{q^{k-1}} & \cdots & g_n^{q^{k-1}} \end{pmatrix}.$$

These codes benefit from an efficient decoding algorithm correcting up to $\left\lfloor \frac{n-k}{2} \right\rfloor$ errors in a deterministic way [8]. They can therefore be used in combination of the McEliece cryptosystem. But the resulting scheme [9] has been attacked due to the strong algebraic structure [22].

Decoding Gabidulin codes. The algorithm employed in order to decode Gabidulin codes has been proposed in [20] and later improved in [4]. Let $\mathcal{G}_{\mathbf{g}}$ denote a Gabidulin code over $\mathbb{F}_{q^m}$ of length n and dimension k generated by the vector $\mathbf{g} \in \mathbb{F}_{q^m}^n$. The decoding problem is stated as follows.

Definition 1.1.9 (**Decoding**$(\mathbf{y}, \mathcal{G}_{\mathbf{g}}, t)$ [20])**.** *Find, if it exists, $\mathbf{c} \in \mathcal{G}_{\mathbf{g}}$ and $\mathbf{e}$ with $\|\mathbf{e}\| \leq t$ such that $\mathbf{y} = \mathbf{c} + \mathbf{e}$.*

One can decode Gabidulin codes using the q-polynomial reconstruction problem:

Definition 1.1.10 (**Reconstruction**$(\mathbf{y}, \mathbf{g}, k, t)$ [20]). *Find a tuple* (V, f) *where* V *is a non-zero* q*-polynomial with* $\deg_q(V) \leq t$ *and* f *is a* q*-polynomial with* $\deg_q(f) < k$ *such that:*

$$V(y_i) = V \circ f(g_i) \text{ with } 1 \leq i \leq n$$

When t is less than the code's decoding capacity $\lfloor (n-k)/2 \rfloor$, the solution of **Reconstruction**$(\mathbf{y}, \mathbf{g}, k, t)$ is unique. Moreover, from a solution of the q-polynomial reconstruction problem, one can get a solution to the decoding problem.

Theorem 1.3 ([20]). *If* (V, f) *is a solution of* **Reconstruction**$(\mathbf{y}, \mathbf{g}, k, t)$*, then* $(\mathbf{c} = f(\mathbf{g}), \mathbf{e} = \mathbf{y} - \mathbf{c})$ *is a solution of* **Decoding**$(\mathbf{y}, \mathcal{G}_{\mathbf{g}}, t)$.

We now consider the linearized variant of the q-polynomial reconstruction problem.

Definition 1.1.11 (**Reconstruction2**$(\mathbf{y}, \mathbf{g}, k, t)$ [20]). *Find a tuple* (V, N) *where* V *is a non-zero* q*-polynomial with* $\deg_q(V) \leq t$ *and* N *is a* q*-polynomial with* $\deg_q(N) \leq k + t - 1$ *such that:*

$$V(y_i) = N(g_i) \text{ with } 1 \leq i \leq n$$

When t is less than the code's decoding capacity $\lfloor (n-k)/2 \rfloor$, the two reconstruction problems are equivalent.

Theorem 1.4 ([20]). *If* (V, f) *is a solution of* **Reconstruction**$(\mathbf{y}, \mathbf{g}, k, t)$*, then* $(V, V \circ f)$ *is a solution of* **Reconstruction2**$(\mathbf{y}, \mathbf{g}, k, t)$.

If $t \leq \lfloor (n-k)/2 \rfloor$ *and if* (V, N) *is a solution of* **Reconstruction2**$(\mathbf{y}, \mathbf{g}, k, t)$*, then* (V, f) *with* f *defined as the quotient of* N *by* V *by left euclidean division in the ring of* q*-polynomials is a solution of* **Reconstruction**$(\mathbf{y}, \mathbf{g}, k, t)$.

The algorithm described in [4] (see Section 4, Algorithm 5) can be used in order to solve the **Reconstruction2**$(\mathbf{y}, \mathbf{g}, k, t)$ problem. This algorithm can be decomposed in two steps:

1. *Initialization step*: Two pairs of q-polynomials (N_0, V_0) and (N_1, V_1) satisfying $V_0(y_i) = N_0(g_i)$ and $V_1(y_i) = N_1(g_i)$ for $1 \leq i \leq k$ are computed. To do so, N_0 is defined as the annihilator q-polynomial on $g_1, \ldots, g_k$, N_1 is defined as the q-polynomial interpolating $y_1, \ldots, y_k$ on $g_1, \ldots, g_k$ while $V_0(X) = 0$ and $V_1(X) = X$.

2. *Interpolation step*: Iteratively, the q-degrees of (N_0, V_0) and (N_1, V_1) are increased using a recurrence relation ensuring that if the interpolation conditions $V(y_i) = N(g_i)$ for $1 \leq i \leq j$ are satisfied at step j, then they are also satisfied at step $j+1$. Besides, this construction ensures that at least one of the pairs satisfies the final degree conditions $\deg_q(V) \leq t$ and $\deg_q(N) \leq k + t - 1$ when the q-polynomials are initialized according to the aforementioned Initialization step.

The correctness of this algorithm is provided in [4], Theorem 10. Using a solution of **Reconstruction2**$(\mathbf{y}, \mathbf{g}, k, t)$, one can solve **Decoding**$(\mathbf{y}, \mathcal{G}_{\mathbf{g}}, t)$ for Gabidulin codes as follows:

Definition 1.1.12 (**Algorithm for Decoding**$(\mathbf{y}, \mathcal{G}_{\mathbf{g}}, t)$ [20, 4])**.**

1. *Find a solution* (V, N) *of* **Reconstruction2**$(\mathbf{y}, \mathbf{g}, k, t)$
2. *Find* f *by computing the left euclidean division of* N *by* V
3. *Retrieve the codeword* $\mathbf{c}$ *by evaluating* f *in* $\mathbf{g}$

Theorem 1.5 ([4])**.** *The complexity of solving* **Decoding**$(\mathbf{y}, \mathcal{G}_{\mathbf{g}}, t)$ *by using the algorithm described in Definition 1.1.12 is* $\mathcal{O}(n^2)$ *operations in* $\mathbb{F}_{q^m}$*.*

Note: Gabidulin decoding has been implemented using the "Polynomials with lower degree" optimization; see [4], Section 4.4.2 for additional details.

1.1.4 Difficult problems for cryptography

In this section, we describe difficult problems which can be used for cryptography and discuss their hardness.

All problems are variants of the *decoding problem*, which consists of looking for the closest codeword to a given vector: when dealing with linear codes, it is readily seen that the decoding problem stays the same when one is given the *syndrome* of the received vector rather than the received vector. We therefore speak of (rank) *Syndrome Decoding* (RSD).

Definition 1.1.13 (RSD Distribution)**.** *For positive integers,* n*,* k*, and* w*, the* $\mathsf{RSD}(n, k, w)$ Distribution *chooses* $\mathbf{H} \xleftarrow{\$} \mathbb{F}_{q^m}^{(n-k)\times n}$ *and* $\mathbf{x} \xleftarrow{\$} \mathbb{F}_{q^m}^{n}$ *such that* $\|\mathbf{x}\| = w$*, and outputs* $(\mathbf{H}, \sigma(\mathbf{x}) = \mathbf{H}\mathbf{x}^\top)$*.*

Definition 1.1.14 (Search RSD Problem)**.** *On input* $(\mathbf{H}, \mathbf{y}^\top) \in \mathbb{F}_{q^m}^{(n-k)\times n} \times \mathbb{F}_{q^m}^{(n-k)}$ *from the RSD distribution, the* Rank Syndrome Decoding Problem $\mathsf{RSD}(n, k, w)$ *asks to find* $\mathbf{x} \in \mathbb{F}_{q^m}^{n}$ *such that* $\mathbf{H}\mathbf{x}^\top = \mathbf{y}^\top$ *and* $\|\mathbf{x}\| = w$*.*

The RSD problem has recently been proven difficult with a probabilistic reduction to the Hamming setting in [13]. For cryptography we also need a decision version of the problem, which is given in the following definition.

Definition 1.1.15 (Decision RSD Problem)**.** *On input* $(\mathbf{H}, \mathbf{y}^\top) \in \mathbb{F}_{q^m}^{(n-k)\times n} \times \mathbb{F}_{q^m}^{(n-k)}$*, the* Decision RSD Problem $\mathsf{DRSD}(n, k, w)$ *asks to decide with non-negligible advantage whether* $(\mathbf{H}, \mathbf{y}^\top)$ *came from the* $\mathsf{RSD}(n, k, w)$ *distribution or the uniform distribution over* $\mathbb{F}_{q^m}^{(n-k)\times n} \times \mathbb{F}_{q^m}^{(n-k)}$*.*

Finally, as our cryptosystem will use ideal codes, we explicitly define the problem on which our cryptosystem will rely. The following definitions describe the DRSD problem in the ideal configuration, and are just a combination of Definition 1.1.5 and 1.1.15. Ideal codes are very useful in cryptography since their compact description allows to decrease considerably the size of the keys.

Definition 1.1.16 (s-IRSD Distribution)**.** *Let $P \in \mathbb{F}_q[X]$ an irreducible polynomial of degree n. For positive integers n, w and s, let $S(n, w, s)$ be the set of the parity-check matrices $\mathbf{H}$ under systematic form of s-ideal codes of type $[sn, n]$ (see Equation 1). The* s-$\mathsf{IRSD}(n, w)$ Distribution *chooses uniformly at random a matrix $\mathbf{H} \xleftarrow{\$} S(n, w, s)$ together with a vector $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_s) \xleftarrow{\$} \mathbb{F}_{q^m}^{sn}$ such that $\|\mathbf{x}\| = w$ and outputs $(\mathbf{H}, \mathbf{H}\mathbf{x}^\top)$.*

Definition 1.1.17 ((Search) s-IRSD Problem)**.** *Let $P \in \mathbb{F}_q[X]$ an irreducible polynomial of degree n. For positive integers n, w, s, a random parity check matrix $\mathbf{H}$ under systematic form of an s-ideal code $\mathcal{C}$ and $\mathbf{y} \xleftarrow{\$} \mathbb{F}_{q^m}^{sn-n}$, the* Search s-ideal RSD Problem s-$\mathsf{IRSD}(n, w)$ *asks to find $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_s) \in \mathbb{F}_{q^m}^{sn}$ such that $\|\mathbf{x}\| = w$ and $\mathbf{y} = \mathbf{x}\mathbf{H}^\top$.*

Assumption 1. *Although there is no general complexity result for ideal codes, decoding these codes is considered as hard by the community. For the rank metric, there is no known generic attack which exploits the ideal structure of the code. Thus, in practice, the best attacks are the same as those for arbitrary codes.*

The problem has a decisional form:

Definition 1.1.18 (Decision s-IRSD Problem)**.** *Let $P \in \mathbb{F}_q[X]$ an irreducible polynomial of degree n. For positive integers n, w and s, let $S(n, w, s)$ be the set of the parity-check matrices $\mathbf{H}$ under systematic form of s-ideal codes of type $[sn, n]$ (see Equation 1). The* Decision s-Ideal RSD Problem s-$\mathsf{DIRSD}(n, w)$ *asks to decide with non-negligible advantage whether $(\mathbf{H}, \mathbf{y}^\top)$ came from the s-$\mathsf{IRSD}(n, w)$ distribution or the uniform distribution over $S(n, w, s) \times \mathbb{F}_{q^m}^{(sn-n)}$.*

As for the ring-LPN problem, there is no known reduction from the search version of s-IRSD problem to its decision version. The proof of [2] cannot be directly adapted in the ideal case, however the best known attacks on the decision version of the problem s-IRSD remain the direct attacks on the search version of the problem s-IRSD.

1.1.5 Encryption and security

Encryption Scheme. An encryption scheme is a tuple of four polynomial time algorithms $(\mathsf{Setup}, \mathsf{KeyGen}, \mathsf{Encrypt}, \mathsf{Decrypt})$:

- $\mathsf{Setup}(1^\lambda)$, where λ is the security parameter, generates the global parameters param of the scheme;
- $\mathsf{KeyGen}(\mathsf{param})$ outputs a pair of keys, a (public) encryption key pk and a (private) decryption key sk;
- $\mathsf{Encrypt}(\mathsf{pk}, \mathbf{m}, \theta)$ outputs a ciphertext $\mathbf{c}$, from the message $\mathbf{m}$, under the encryption key pk using randomness θ;
- $\mathsf{Decrypt}(\mathsf{sk}, \mathbf{c})$ outputs the plaintext $\mathbf{m}$, encrypted in the ciphertext $\mathbf{c}$ or $\perp$.

Such an encryption scheme has to satisfy both *Correctness* and *Indistinguishability under Chosen Plaintext Attack* (IND-CPA) security properties.

Correctness: For every λ, every $\mathsf{param} \leftarrow \mathsf{Setup}(1^\lambda)$, every pair of keys $(\mathsf{pk}, \mathsf{sk})$ generated by KeyGen, every message $\mathbf{m}$, we should have $P[\mathsf{Decrypt}(\mathsf{sk}, \mathsf{Encrypt}(\mathsf{pk}, \mathbf{m}, \theta)) = \mathbf{m}] = 1 - \mathsf{negl}(\lambda)$ for $\mathsf{negl}(\cdot)$ a negligible function, where the probability is taken over varying randomness.

IND-CPA [15]: This notion formalized by the game depicted in Fig. 1, states that an adversary should not be able to efficiently guess which plaintext has been encrypted even if he knows it is one among two plaintexts of his choice.

In the following, we denote by $|\mathcal{A}|$ the running time of an adversary $\mathcal{A}$. The global advantage for polynomial time adversaries running in time less than t is:

$$\mathsf{Adv}^{\mathsf{ind}}_{\mathcal{E}}(\lambda, t) = \max_{|\mathcal{A}| \leq t} \mathsf{Adv}^{\mathsf{ind}}_{\mathcal{E},\mathcal{A}}(\lambda), \tag{3}$$

where $\mathsf{Adv}^{\mathsf{ind}}_{\mathcal{E},\mathcal{A}}(\lambda)$ is the advantage the adversary $\mathcal{A}$ has in winning game $\mathbf{Exp}^{\mathsf{ind}-b}_{\mathcal{E},\mathcal{A}}(\lambda)$:

$\mathbf{Exp}^{\mathsf{ind}-b}_{\mathcal{E},\mathcal{A}}(\lambda)$
1. $\mathsf{param} \leftarrow \mathsf{Setup}(1^\lambda)$
2. $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(\mathsf{param})$
3. $(\mathbf{m}_0, \mathbf{m}_1) \leftarrow \mathcal{A}(\texttt{FIND} : \mathsf{pk})$
4. $\mathbf{c}^* \leftarrow \mathsf{Encrypt}(\mathsf{pk}, \mathbf{m_b}, \theta)$
5. $b' \leftarrow \mathcal{A}(\texttt{GUESS} : \mathbf{c}^*)$
6. `RETURN` b'

Figure 1: Game for the IND-CPA security of an asymmetric encryption scheme.

$$\mathsf{Adv}^{\mathsf{ind}}_{\mathcal{E},\mathcal{A}}(\lambda) = \left|\Pr[\mathbf{Exp}^{\mathsf{ind}-1}_{\mathcal{E},\mathcal{A}}(\lambda) = 1] - \Pr[\mathbf{Exp}^{\mathsf{ind}-0}_{\mathcal{E},\mathcal{A}}(\lambda) = 1]\right|. \tag{4}$$

IND-CPA and IND-CCA2: Note that the standard security requirement for a public key cryptosystem is IND-CCA2, *indistinguishability against adaptive chosen-ciphertext attacks*, and not just IND-CPA. The main difference is that for IND-CCA2 indistinguishability must hold even if the attacker is given a *decryption oracle* first when running the `FIND` algorithm and also when running the `GUESS` algorithm (but cannot query the oracle on the challenge ciphertext $\boldsymbol{c}^*$). We do not present the associated formal game and definition as an existing (and inexpensive) transformation can be used [17] for our scheme to pass from IND-CPA to IND-CCA2.

In [17] Hofheinz et al. present a generic transformation that takes into account decryption errors and can be applied directly to our scheme. Roughly, their construction provides a way to convert a guarantee against passive adversaries into indistinguishability

against active ones by turning a public key cryptosystem into a KEM-DEM. The tightness (the quality factor) of the reduction depends on the ciphertext distribution. Regarding our scheme, random words only have a negligible (in the security parameter) probability of being valid ciphertexts. In other words, the γ-spreadness factor of [17] is small enough so that there is no loss between the IND-CPA security of our public key cryptosystem and the IND-CCA2 security of the KEM-DEM version.

The security reduction is tight in the random oracle model and does not require any supplemental property from our scheme as we have the IND-CPA property (instead of just a weaker property called *One-Wayness*). Let us denote by $\mathsf{Encrypt}(\mathsf{pk}, \mathbf{m}, \theta)$ the encryption function defined in Fig. 2 that uses randomness θ to generate uniformly random values $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{e}$. The idea of [17] transformation is to de-randomize the encryption function $\mathsf{Encrypt}(\mathsf{pk}, \mathbf{m}, \theta)$ by using a hash function $\mathcal{G}$ and do a deterministic encryption of $\mathbf{m}$ by calling $c = \mathsf{Encrypt}(\mathsf{pk}, \mathbf{m}, \mathcal{G}(\mathbf{m}))$. The ciphertext is sent together with a hash $K = \mathcal{H}(\mathbf{c}, \mathbf{m})$ that ties the ciphertext to the plaintext. The receiver then decrypts $\mathbf{c}$ into $\mathbf{m}$, checks the hash value, and uses again the deterministic encryption to check that $\mathbf{c}$ is indeed *the* ciphertext associated to $\mathbf{m}$.

As the reduction is tight we do not need to change our parameters when we pass from IND-CPA to IND-CCA2. From a computational point of view, the overhead for the sender is two hash calls and for the receiver it is two hash calls and an encrypt call. From a communication point of view the overhead is the bitsize of a hash (or two if the reduction must hold in the Quantum Random Oracle Model, see [17] for more details).

1.2 Presentation of the scheme

In this section, we describe our proposal: RQC. We begin with the PKE version (RQC.PKE), then describe the transformation of [17] to obtain a KEM-DEM that achieves IND-CCA2 (RQC.KEM). Finally, we discuss an hybrid encryption scheme using NIST standard conversion techniques (RQC.HE). Parameter sets can be found in Sec. 1.4.

Notations: $\mathcal{S}_w^n(\mathbb{F}_{q^m})$ stands for the set of vectors of length n and rank weight w over $\mathbb{F}_{q^m}$ and $\mathcal{S}_{1,w}^n(\mathbb{F}_{q^m})$ stands for the set of vectors of length n of rank weight w, *such that its support contains* 1:

$$\mathcal{S}_w^n(\mathbb{F}_{q^m}) = \{\mathbf{x} \in \mathbb{F}_{q^m}^n : \dim(\mathrm{Supp}(\mathbf{x})) = w\}$$
$$\mathcal{S}_{1,w}^n(\mathbb{F}_{q^m}) = \{\mathbf{x} \in \mathbb{F}_{q^m}^n : \dim(\mathrm{Supp}(\mathbf{x})) = w, 1 \in \mathrm{Supp}(\mathbf{x})\}$$

1.2.1 Public key encryption version (RQC.PKE)

Presentation of the scheme. RQC uses two types of codes: a Gabidulin code $\mathcal{G}_{\mathbf{g}}(n, k, m)$ generated by $\mathbf{G} \in \mathbb{F}_{q^m}^{k \times n}$ which can correct at least δ errors via an efficient algorithm $\mathcal{G}_{\mathbf{g}}.\mathsf{Decode}(\cdot)$; and a random ideal $[2n, n]$ code, of parity-check matrix $(\mathbf{1}, \mathbf{h})$. The four polynomial-time algorithms constituting our scheme are depicted in Fig. 2.

- $\mathsf{Setup}(1^\lambda)$: generates and outputs the global parameters $\mathsf{param} = (n, k, \delta, w, w_r, P)$ where $P \in \mathbb{F}_q[X]$ is an irreducible polynomial of degree n.
- $\mathsf{KeyGen}(\mathsf{param})$: samples $\mathbf{h} \xleftarrow{\$} \mathbb{F}_{q^m}^n$, $\mathbf{g} \xleftarrow{\$} \mathcal{S}_n^n(\mathbb{F}_{q^m})$ and $(\mathbf{x}, \mathbf{y}) \xleftarrow{\$} \mathcal{S}_{1,w}^{2n}(\mathbb{F}_{q^m})$, computes the generator matrix $\mathbf{G} \in \mathbb{F}_{q^m}^{k \times n}$ of $\mathcal{G}_{\mathbf{g}}(n, k, m)$, sets $\mathsf{pk} = (\mathbf{g}, \mathbf{h}, \mathbf{s} = \mathbf{x} + \mathbf{h} \cdot \mathbf{y} \mod P)$ and $\mathsf{sk} = (\mathbf{x}, \mathbf{y})$, returns $(\mathsf{pk}, \mathsf{sk})$.
- $\mathsf{Encrypt}(\mathsf{pk}, \mathbf{m}, \theta)$: uses randomness θ to generate $(\mathbf{e}, \mathbf{r}_1, \mathbf{r}_2) \xleftarrow{\$} \mathcal{S}_{w_r}^{3n}(\mathbb{F}_{q^m})$, sets $\mathbf{u} = \mathbf{r}_1 + \mathbf{h} \cdot \mathbf{r}_2 \mod P$ and $\mathbf{v} = \mathbf{mG} + \mathbf{s} \cdot \mathbf{r}_2 + \mathbf{e} \mod P$, returns $\mathbf{c} = (\mathbf{u}, \mathbf{v})$.
- $\mathsf{Decrypt}(\mathsf{sk}, \mathbf{c})$: returns $\mathcal{G}_{\mathbf{g}}.\mathsf{Decode}(\mathbf{v} - \mathbf{u} \cdot \mathbf{y} \mod P)$.

Figure 2: Description of our proposal RQC.PKE.

Notice that the generator matrix $\mathbf{G}$ of the code $\mathcal{G}_{\mathbf{g}}$ is publicly known, so the security of the scheme and the ability to decrypt do not rely on the knowledge of the error correcting code $\mathcal{G}_{\mathbf{g}}$ being used.

Correctness. The correctness of our encryption scheme clearly relies on the decoding capability of the code $\mathcal{G}_{\mathbf{g}}$. Specifically, assuming $\mathcal{G}_{\mathbf{g}}.\mathsf{Decode}$ correctly decodes $\mathbf{v} - \mathbf{u} \cdot \mathbf{y}$, we have:

$$\mathsf{Decrypt}\left(\mathsf{sk}, \mathsf{Encrypt}\left(\mathsf{pk}, \mathbf{m}, \theta\right)\right) = \mathbf{m}. \tag{5}$$

And $\mathcal{G}_{\mathbf{g}}.\mathsf{Decode}$ correctly decodes $\mathbf{v} - \mathbf{u} \cdot \mathbf{y}$ whenever

$$\|\mathbf{s} \cdot \mathbf{r}_2 - \mathbf{u} \cdot \mathbf{y} + \mathbf{e}\| \leq \delta \tag{6}$$
$$\|\left(\mathbf{x} + \mathbf{h} \cdot \mathbf{y}\right) \cdot \mathbf{r}_2 - \left(\mathbf{r}_1 + \mathbf{h} \cdot \mathbf{r}_2\right) \cdot \mathbf{y} + \mathbf{e}\| \leq \delta \tag{7}$$
$$\|\mathbf{x} \cdot \mathbf{r}_2 - \mathbf{r}_1 \cdot \mathbf{y} + \mathbf{e}\| \leq \delta \tag{8}$$

There is no decryption failure, or to be more accurate, the probability that a decryption failure occurs is null. More details are provided at the beginning of Sec. 1.4.

1.2.2 KEM/DEM version (RQC.KEM)

Let $\mathcal{E}$ be an instance of the RQC cryptosystem as described above. Let $\mathcal{G}$, $\mathcal{H}$, and $\mathcal{K}$ be hash functions, typically SHA512 as advised by NIST[1]. The KEM-DEM version of the RQC cryptosystem is described in Figure 3.

According to [17], the KEM-DEM version of RQC is IND-CCA2. More details regarding the tightness of the reduction are provided at the end of Sec. 1.4.

Security concerns and implementation details. Notice that while NIST only recommends SHA512 as a hash function (or TupleHash256 for hardware efficiency purposes), the transformation of [17] would be dangerous – at least in our setting – if one sets $\mathcal{G} = \mathcal{H}$.

[1]See Dustin Moody's mail entitled "new FAQ question" on PQC-forum (20/07/2017 – 12:58)

- $\mathsf{Setup}(1^\lambda)$: as before, except that the plaintext space has size $k \times m \geq 256$ as required by NIST.
- $\mathsf{KeyGen}(\mathsf{param})$: exactly as before.
- $\mathsf{Encapsulate}(\mathsf{pk})$: generate $\mathbf{m} \xleftarrow{\$} \mathbb{F}_{q^m}^k$ (this will serve as a seed to derive the shared key). Derive the randomness $\theta \leftarrow \mathcal{G}(\mathbf{m})$. Generate the ciphertext $\mathbf{c} \leftarrow (\mathbf{u}, \mathbf{v}) = \mathcal{E}.\mathsf{Encrypt}(\mathsf{pk}, \mathbf{m}, \theta)$, and derive the symmetric key $K \leftarrow \mathcal{K}(\mathbf{m}, \mathbf{c})$. Let $\mathbf{d} \leftarrow \mathcal{H}(\mathbf{m})$, and send $(\mathbf{c}, \mathbf{d})$.
- $\mathsf{Decapsulate}(\mathsf{sk}, \mathbf{c}, \mathbf{d})$: Decrypt $\mathbf{m}' \leftarrow \mathcal{E}.\mathsf{Decrypt}(\mathsf{sk}, \mathbf{c})$, compute $\theta' \leftarrow \mathcal{G}(\mathbf{m}')$, and (re-)encrypt $\mathbf{m}'$ to get $\mathbf{c}' \leftarrow \mathcal{E}.\mathsf{Encrypt}(\mathsf{pk}, \mathbf{m}', \theta')$. If $\mathbf{c} \neq \mathbf{c}'$ or $\mathbf{d} \neq \mathcal{H}(\mathbf{m}')$ then abort. Otherwise, derive the shared key $K \leftarrow \mathcal{K}(\mathbf{m}, \mathbf{c})$.

Figure 3: Description of our proposal RQC.KEM.

Indeed, publishing the randomness $\theta = \mathcal{G}(\mathbf{m}) = \mathcal{H}(\mathbf{m}) = \mathbf{d}$ used to generate $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{e}$, would allow one to retrieve $\mathbf{s}$, the secret key of $\mathcal{E}$.

We therefore suggest to use a pseudo-random function for $\mathcal{G}$, such as an AES-based seed expander, and SHA512 for $\mathcal{H}$.

1.2.3 A hybrid encryption scheme (RQC.HE)

While NIST claimed that they will be using generic transformations to convert any IND-CCA2 KEM into an IND-CCA2 PKE, no detail on these conversions have been provided. We therefore refer to RQC.HE to designate the PKE scheme resulting from applying a generic conversion to RQC.KEM.

1.3 Representation of objects

Field elements. Elements of $\mathbb{F}_{q^m}$ are represented as vectors of size m over $\mathbb{F}_q$. For RQC, q is always choosen equal to 2 (see section 1.4) thus $e \in \mathbb{F}_{q^m}$ is represented as $(e_0, \ldots, e_{m-1}) \in \mathbb{F}_2^m$. Elements are stored using $\lceil m/8 \rceil$ bytes in which the unused $8 \times \lceil m/8 \rceil - m$ bits are zero-padded. The first bit e_0 corresponds to the constant coefficient of the polynomial e.

Vectors. Elements of $\mathbb{F}_{q^m}^n$ are represented as n-dimensional arrays of $\mathbb{F}_{q^m}$ elements.

Seeds. The considered seedexpander has been provided by the NIST. It is initialized with a byte string of length 40 of which 32 are used as the `seed` and 8 are used as the `diversifier`. In addition, it is initialized with `max_length` equal to $2^{32} - 1$.

1.3.1 Parsing vectors from/to byte strings

Vectors of $\mathbb{F}_{q^m}^n$ are converted to byte strings using either a compact representation or a standard one. In the standard representation, a vector is seen as the concatenation of its elements thus leading to a $n\lceil m/8 \rceil$ long byte string. In the compact representation, the unused bits of each element are removed thus leading to a $\lceil nm/8 \rceil$ long byte string. The compact representation is used for the public key pk, the secret key sk, the ciphertext $\mathbf{c}$ and for $\mathbf{m}$ as input of the hash function $\mathcal{G}$ when generating θ. The standard representation is used for the inputs of the hash functions $\mathcal{H}$ and $\mathcal{K}$.

1.3.2 Keys and ciphertext representation

The secret key $\mathsf{sk} = (\mathbf{x}, \mathbf{y})$ is represented as $\mathsf{sk} = (\mathbf{seed1})$ where **seed1** is used to generate $\mathbf{x}$ and $\mathbf{y}$. The public key $\mathsf{pk} = (\mathbf{g}, \mathbf{h}, \mathbf{s})$ is represented as $\mathsf{pk} = (\mathbf{seed2}, \mathbf{s})$ where **seed2** is used to generate $\mathbf{g}$ and $\mathbf{h}$. The generator matrix $\mathbf{G}$ is generated from $\mathbf{g}$ with respect to Definition 1.1.8. The ciphertext $\mathbf{c}$ is represented as $(\mathbf{u}, \mathbf{v}, \mathbf{d})$ where $\mathbf{d}$ is generated using SHA512. The secret key has size 40 bytes, the public key has size $40 + \lceil nm/8 \rceil$ bytes and the ciphertext has size $2\lceil nm/8 \rceil + 64$ bytes.

1.3.3 Randomness and vector generation

Random bytes are generated using the NIST provided `randombytes` or `seedexpander` functions. The `randombytes` function is used to generate **seed1** and **seed2** as well as $\mathbf{m}$. The `seedexpander` function is used to generate θ (using $\mathbf{m}$ as seed) as well as $\mathbf{x}$, $\mathbf{y}$ (using **seed1** as seed), $\mathbf{g}$, $\mathbf{h}$ (using **seed2** as seed) and $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{e}$ (using θ as seed).

Random vectors are sampled uniformly from $\mathbb{F}_{q^m}^k$, $\mathbb{F}_{q^m}^n$, $\mathcal{S}_n^n(\mathbb{F}_{q^m})$, $\mathcal{S}_{1,w}^{2n}(\mathbb{F}_{q^m})$ or $\mathcal{S}_{w_r}^{3n}(\mathbb{F}_{q^m})$. Sampling from $\mathbb{F}_{q^m}^k$ and $\mathbb{F}_{q^m}^n$ is performed by filling the mathematical representation of the vector with random bits. Sampling from $\mathcal{S}_n^n(\mathbb{F}_{q^m})$ uses the same process and repeat it until a full rank vector is found. Sampling from $\mathcal{S}_{1,w}^{2n}(\mathbb{F}_{q^m})$ and $\mathcal{S}_{w_r}^{3n}(\mathbb{F}_{q^m})$ starts by generating a full rank support vector of size $w - 1$ (appending 1 to it) or size w_r. Next, the sampled vector is generated by setting the coordinates of the support vector at random positions and setting the remaining coordinates as random linear combinations of the support vector coordinates.

1.4 Parameters

Error distribution and decoding algorithm: no decryption failure. The case of the rank metric is much simpler than for the Hamming metric. Indeed in that case the decryption algorithm of our cryptosystem asks to decode an error $\mathbf{e}' = \mathbf{x} \cdot \mathbf{r}_2 - \mathbf{r}_1 \cdot \mathbf{y} + \mathbf{e}$ where the words $\mathbf{x}$ and $\mathbf{y}$ (resp. $\mathbf{r}_1$ and $\mathbf{r}_2$) have rank weight w (resp. w_r). Unlike the Hamming metric weight, the rank weight of the vector $\mathbf{x} \cdot \mathbf{r}_2 - \mathbf{r}_1 \cdot \mathbf{y}$ is almost always ww_r and is in any case bounded from above by ww_r. In particular, with a strong probability,

the rank weight of $\mathbf{x} \cdot \mathbf{r}_2 - \mathbf{r}_1 \cdot \mathbf{y}$ is the same as the rank weight of $\mathbf{x} \cdot \mathbf{r}_2$ since $\mathbf{x}$ and $\mathbf{y}$ share the same rank support, as do $\mathbf{r}_1$ and $\mathbf{r}_2$. We consider the additional error $\mathbf{e}$ of rank w_r with same error support as $\mathbf{r}_1$ and $\mathbf{r}_2$. So that overall the error $\mathbf{e}'$ to decode for decryption has a rank weight upper bounded by $(w+1)w_r$.

Now since we choose the secret vector $(\mathbf{x}, \mathbf{y})$ such that its support is a random subspace of $\mathbb{F}_{q^m}$ of dimension w containing 1, the weight of $\mathbf{e}'$ is upper bounded by ww_r. Indeed, in this case, the support of $\mathbf{e}$ is included in the product of the supports of $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{r}_1, \mathbf{r}_2)$. This does not modify the security proof, and impacts only the value of w in the choice of parameters.

For decoding, we consider Gabidulin $[n, k]$ codes over $\mathbb{F}_{q^m}$, which can decode $\frac{n-k}{2}$ rank errors and choose our parameters such that $ww_r \leq \frac{n-k}{2}$, so that, *there is no decryption failure*.

Parameters and tightness of the reduction. The practical security of the scheme relies on the 2-DIRSD problem for the public key, for a small weight vector of weight $w = \|\mathbf{x}\| = \|\mathbf{y}\|$ with $w = \mathcal{O}(\sqrt{n})$. The IND-CPA security of the scheme could be reduced to the 3-DIRSD problem, decoding a random ideal $[3n, n]$ code for a small weight vector $(\mathbf{r}_1, \mathbf{e}, \mathbf{r}_2)$. In the proof, the error vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ share the same error support E of dimension w_r, for the encryption part the error support of $\mathbf{e}$ can also be taken as E, so that the problem is tightly reduced to the 3-DIRSD problem for rank metric with weight w_r, since all three vectors $\mathbf{r}_1, \mathbf{r}_2$ and $\mathbf{e}$ have the same error support E of dimension w_r. In that case the attacker wants to decode a $[3n, n]$ rank metric code, the best known attack is described in [12, 3]. Since on one hand the attacker wants to attack a length $2n$ code and on the other hand to attack a length $3n$ code, which is easier, we consider different weights for the secret key $\mathbf{x}, \mathbf{y}$ of weight w and for the random chosen values for the encryption $\mathbf{r}_1, \mathbf{e}, \mathbf{r}_2$ of weight w_r, typically we chose $w \approx \frac{2}{3}w_r$. For the secret key, we consider $1 \in \mathrm{Supp}(\mathbf{x}, \mathbf{y})$, now since finding a small weight codeword of weight w with support containing 1 is harder than finding a small weight vector of weight $w-1$, we consider $w-1$ for the security reduction to the 2-DIRSD problem, and the weight w_r is chosen according to the 3-DIRSD problem and the best known attacks of [12, 3], whose complexity is given in Section 5. The best quantum attacks on the rank metric problems follow [11], in that case there is square root gain on the probabilistic part of the attack (details are given in [11]).

Choice of parameters: overall the parameters proposed in Tab. 1 correspond to tight reduction for generic instances of the 2-DIRSD and 3-DIRSD problems in the rank metric. Parameters are chosen such that $1 \in \mathrm{Supp}(\mathbf{x}, \mathbf{y})$, the vectors $\mathbf{r}_1, \mathbf{r}_2$ and $\mathbf{e}$ have the same random support of dimension w_r.

All of our submissions use ideal code over $\mathbb{F}_{2^m}$ in order to reduce the size of the key and to allow to compute the syndrome of an error as sums and products of polynomials in $\mathbb{F}_{2^m}[X]/\langle P \rangle$, with $P \in \mathbb{F}_2[X]$ of degree n. In order to avoid folding attacks (see [16]), P is chosen irreducible. Moreover, to decrease the computational costs, we want P to be sparse. We have obtained these polynomials with the Magma software. More details are available at http://magma.maths.usyd.edu.au/magma/handbook/text/193#1685. Tab. 2 presents the aforementioned polynomials as well as the polynomials used to define $\mathbb{F}_{2^m}$.

The decoding Gabidulin code has length n, dimension k over $\mathbb{F}_{q^m}$ and corrects errors of weight up to $(n-k)/2 \geqslant ww_r$. The resulting public key, secret key, ciphertext and shared secret sizes are given in Tab. 3. The aforementioned sizes are the ones used in our reference implementation except that we also concatenate the public key within the secret key in order to respect the NIST API.

	RQC Cryptosystem Parameters						
Instance	q	m	n	k	w	w_r	Security
RQC-I	2	97	67	4	5	6	128
RQC-II	2	107	101	3	6	8	192
RQC-III	2	137	131	3	7	9	256

Table 1: Parameter sets for RQC. The security is expressed in bits.

Instance	P	Π
RQC-I	$X^{67}+X^5+X^2+X+1$	$X^{97}+X^6+1$
RQC-II	$X^{101}+X^7+X^6+X+1$	$X^{107}+X^9+X^7+X^4+1$
RQC-III	$X^{131}+X^8+X^3+X^2+1$	$X^{137}+X^{21}+1$

Table 2: Polynomials considered for RQC. P is the polynomial used to define $\mathbb{F}_{q^m}^n$ as $\mathbb{F}_{q^m}[X]/\langle P\rangle$ and Π is the polynomial used to define $\mathbb{F}_{q^m}$ as $\mathbb{F}_q[X]/\langle\Pi\rangle$.

Instance	pk size	sk size	ct size	ss size	Security
RQC-I	853	40	1690	64	128
RQC-II	1391	40	2766	64	192
RQC-III	2284	40	4552	64	256

Table 3: Sizes in bytes for RQC (see section 1.3). The security is expressed in bits.

2 Performance Analysis

In this section, we provide concrete performance measures of our implementation. For each parameter set, results have been obtained by running 100,000 random instances and computing their average execution time. The benchmarks have been performed on a machine

running Archlinux. The latter has 16GB of memory and an Intel® Core™ i7-7820X CPU @ 3.6GHz for which the Hyper-Threading, Turbo Boost and SpeedStep features were disabled. The scheme have been compiled with `g++` (version 8.2.1) using the compilation flags `-O3 -pedantic -pthread`. The following third party libraries have been used: `openssl` (version 1.1.1b), `gmp` (version 6.1.2), `NTL` (version 11.3.2) [24] and `GF2X` (version 1.2).

2.1 Reference Implementation

The performances of our reference implementation on the aforementioned benchmark platform are described in Tab. 4 (timings in ms) and Tab. 5 (millions of CPU cycles required). The reference implementation provided for round 2 relies on the NTL library [24] in order to implement finite field operations rather than the MPFQ library [14] as previously. Although NTL might be outperformed by MPFQ for operations in $\mathbb{F}_{q^m}$, NTL outperforms MPFQ for the multiplication over $\mathbb{F}_{q^m}^n$ thus improving the overall performances of RQC. As the cryptosystem algorithms are loosely coupled with the underlying finite field operations, one can easily switch between various implementations for the finite field arithmetic.

Instance	KeyGen	Encrypt	Decrypt
RQC-128	0.19	0.36	1.85
RQC-192	0.31	0.60	4.06
RQC-256	0.51	0.99	6.43

Table 4: Timings (in ms) of RQC reference implementation.

Instance	KeyGen	Encrypt	Decrypt
RQC-128	0.70	1.30	6.66
RQC-192	1.12	2.18	14.68
RQC-256	1.82	3.55	23.20

Table 5: Millions of CPU cycles of RQC reference implementation.

2.2 Optimized Implementation

No optimized implementation has been provided. As a consequence, the folders `Optimized_Implementation/` and `Reference_Implementation/` are identical. Some functions might use AVX2 instructions depending on the NTL implementation. An optimized implementation that no longer relies on the NTL library nor the MPFQ library and uses AVX2 instructions to speed-up finite field operations will be provided later.

2.3 Additional Implementations

As Gabidulin code's decoding algorithm is deterministic, it can inherently be implemented in constant time. This can be performed without any performance cost for honest users as explained in [5]. A constant-time implementation based on the results described in section 5.4 will be provided later.

3 Known Answer Test Values

Known Answer Test (KAT) values have been generated using the script provided by the NIST. They are available in the folder `KAT/Reference_Implementation/`. Since the reference and optimized implementations are identical, `KAT/Optimized_Implementation/` is just a copy of `KAT/Reference_Implementation/`.

In addition, we provide, for each parameter set, an example with *intermediate values* in the folder `KAT/Reference_Implementation/`.

Notice that one can generate the aforementioned test files using respectively the `kat` and `verbose` modes of our implementation. The procedure to follow in order to do so is detailed in the technical documentation.

4 Security

In this section we prove the security of our encryption scheme viewed as a PKE scheme (IND-CPA). The security of the KEM/DEM version is provided by the transformation described in [17], and the tightness of the reduction provided by this transformation has been discussed at the end of Sec. 1.1.5.

Theorem 4.1. *The scheme presented above is* IND-CPA *under the* 2*-DIRSD and* 3*-DIRSD assumptions.*

Proof. To prove the security of the scheme, we are going to build a sequence of games transitioning from an adversary receiving an encryption of message $\mathbf{m}_0$ to an adversary receiving an encryption of a message $\mathbf{m}_1$ and show that if the adversary manages to distinguish one from the other, then we can build a simulator breaking the DIRSD assumption for 2-ideal codes 3-ideal codes (codes with parameters $[2n, n]$ or $[3n, n]$), and running in approximately the same time.

Game $\mathbf{G}_1$: This is the real game, which we can state algorithmically as follows:

$\mathbf{Game}^1_{\mathcal{E},\mathcal{A}}(\lambda)$
1. $\mathsf{param} \leftarrow \mathsf{Setup}(1^\lambda)$
2. $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(\mathsf{param})$ with $\mathsf{pk} = (\mathbf{g}, \mathbf{h}, \mathbf{s} = \mathbf{x} + \mathbf{h} \cdot \mathbf{y})$ and $\mathsf{sk} = (\mathbf{x}, \mathbf{y})$
3. $(\mathbf{m}_0, \mathbf{m}_1) \leftarrow \mathcal{A}(\mathtt{FIND} : \mathsf{pk})$

4. $\mathbf{c}^* \leftarrow \mathsf{Encrypt}(\mathsf{pk}, \mathbf{m_0}, \theta)$
5. $\mathbf{b}' \leftarrow \mathcal{A}(\texttt{GUESS} : \mathbf{c}^*)$
6. `RETURN` $\mathbf{b}'$

Game $\mathbf{G}_2$: In this game we start by forgetting the decryption key sk, and taking $\mathbf{s}$ at random, and then proceed honestly:

$\mathbf{Game}^2_{\mathcal{E},\mathcal{A}}(\lambda)$
1. $\mathsf{param} \leftarrow \mathsf{Setup}(1^\lambda)$
2a. $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(\mathsf{param})$ with $\mathsf{pk} = (\mathbf{g}, \mathbf{h}, \mathbf{s} = \mathbf{x} + \mathbf{h} \cdot \mathbf{y})$ and $\mathsf{sk} = (\mathbf{x}, \mathbf{y})$
2b. $\mathbf{s} \xleftarrow{\$} \mathbb{F}_{q^m}^n$
2c. $(\mathsf{pk}, \mathsf{sk}) \leftarrow ((\mathbf{g}, \mathbf{h}, \mathbf{s}), \mathbf{0})$
3. $(\mathbf{m}_0, \mathbf{m}_1) \leftarrow \mathcal{A}(\texttt{FIND} : \mathsf{pk})$
4. $\mathbf{c}^* \leftarrow \mathsf{Encrypt}(\mathsf{pk}, \mathbf{m_0}, \theta)$
5. $\mathbf{b}' \leftarrow \mathcal{A}(\texttt{GUESS} : \mathbf{c}^*)$
6. `RETURN` $\mathbf{b}'$

The adversary has access to pk and $\boldsymbol{c}^*$. As he has access to pk and the $\mathsf{Encrypt}$ function, anything that is computed from pk and $\boldsymbol{c}^*$ can also be computed from just pk. Moreover, the distribution of $\boldsymbol{c}^*$ is independent of the game we are in, and therefore we can suppose the only input of the adversary is pk. Suppose he has an algorithm $\mathcal{D}_\lambda$, taking pk as input, that distinguishes with advantage ϵ Game $\mathbf{G}_1$ and Game $\mathbf{G}_2$, for some security parameter λ. Then he can also build an algorithm $\mathcal{D}'_{\mathcal{E},\mathcal{D}_\lambda}$ which solves the 2-$\mathsf{DIRSD}(n, \omega)$ problem for parameters (n, ω) resulting from $\mathsf{Setup}(1^\lambda)$, with the same advantage ϵ, when given as input a challenge $(\mathbf{H}, \mathbf{y}^\top) \in \mathbb{F}_{q^m}^{n \times 2n} \times \mathbb{F}_{q^m}^n$.

$\mathcal{D}'_{\mathcal{E},\mathcal{D}_\lambda}((\mathbf{H}, \mathbf{y}^\top))$
1. Set $\mathsf{param} \leftarrow \mathsf{Setup}(1^\lambda)$
2a. $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(\mathsf{param})$ with $\mathsf{pk} = (\mathbf{g}', \mathbf{h}', \mathbf{s}' = \mathbf{x}' + \mathbf{h}' \cdot \mathbf{y}')$ and $\mathsf{sk} = (\mathbf{x}', \mathbf{y}')$
2b. $(\mathsf{pk}, \mathsf{sk}) \leftarrow ((\mathbf{g}', \mathbf{h}, \mathbf{y}), \mathbf{0})$
3. $\mathbf{b}' \leftarrow \mathcal{D}_\lambda(\mathsf{pk})$
4. If $\mathbf{b}' == 1$ output `IRSD`
5. If $\mathbf{b}' == 2$ output `UNIFORM`

Note that if we define pk as $(\mathbf{g}', \mathbf{h}, \mathbf{y})$ with $\mathbf{g}'$ generated by $\mathsf{KeyGen}(\mathsf{param})$ and $(\mathbf{H}, \mathbf{y}^\top)$ from a 2-$\mathsf{IRSD}(n, \omega)$ distribution pk follows exactly the same distribution as in Game $\mathbf{G}_1$. On the other hand if $(\mathbf{H}, \mathbf{y}^\top)$ comes from a uniform distribution, then pk follows exactly the same distribution as in Game $\mathbf{G}_2$. Thus we have

$$Pr\left[\mathcal{D}'_{\mathcal{E},\mathcal{D}_\lambda}((\mathbf{H}, \mathbf{y}^\top)) = \texttt{IRSD} | (\mathbf{H}, \mathbf{y}^\top) \leftarrow 2\text{-}\mathsf{IRSD}(n, \omega)\right] = Pr\left[\mathcal{D}_\lambda(\mathsf{pk}) = 1 | \mathsf{pk} \text{ from } \mathbf{Game}^1_{\mathcal{E},\mathcal{A}}(\lambda)\right]$$

and

$$Pr\left[\mathcal{D}'_{\mathcal{E},\mathcal{D}_\lambda}((\mathbf{H}, \mathbf{y}^\top)) = \texttt{UNIFORM} | (\mathbf{H}, \mathbf{y}^\top) \leftarrow 2\text{-}\mathsf{IRSD}(n, \omega)\right] = Pr\left[\mathcal{D}_\lambda(\mathsf{pk}) = 2 | \mathsf{pk} \text{ from } \mathbf{Game}^1_{\mathcal{E},\mathcal{A}}(\lambda)\right]$$

And similarly when $(\mathbf{H}, \mathbf{y}^\top)$ is uniform the probabilities of $\mathcal{D}'_{\mathcal{E},\mathcal{D}_\lambda}$ outputs match those of $\mathcal{D}_\lambda$ when pk is from $\mathbf{Game}^2_{\mathcal{E},\mathcal{A}}(\lambda)$. The advantage of $\mathcal{D}'_{\mathcal{E},\mathcal{D}_\lambda}$ is therefore equal to the advantage of $\mathcal{D}_\lambda$.

Game $\mathbf{G}_3$: Now that we no longer know the decryption key, we can start generating random ciphertexts. So instead of picking correctly weighted $\mathbf{r}_1, \mathbf{r}_2, \mathbf{e}$, the simulator now picks random vectors in the full space.

$\mathbf{Game}^3_{\mathcal{E},\mathcal{A}}(\lambda)$
1. $\mathsf{param} \leftarrow \mathsf{Setup}(1^\lambda)$
2a. $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(\mathsf{param})$ with $\mathsf{pk} = (\mathbf{g}, \mathbf{h}, \mathbf{s} = \mathbf{x} + \mathbf{h} \cdot \mathbf{y})$ and $\mathsf{sk} = (\mathbf{x}, \mathbf{y})$
2b. $\mathbf{s} \xleftarrow{\$} \mathbb{F}_{q^m}^n$
2c. $(\mathsf{pk}, \mathsf{sk}) \leftarrow ((\mathbf{g}, \mathbf{h}, \mathbf{s}), \mathbf{0})$
3. $(\mathbf{m}_0, \mathbf{m}_1) \leftarrow \mathcal{A}(\texttt{FIND} : \mathsf{pk})$
4a. Use randomness θ to generate $\mathbf{e} \xleftarrow{\$} \mathbb{F}_{q^m}^n$, $\mathbf{r} = (\mathbf{r}_1, \mathbf{r}_2) \xleftarrow{\$} \mathbb{F}_{q^m}^{2n}$ uniformly at random
4b. $\mathbf{u} \leftarrow \mathbf{r}_1 + \mathbf{h} \cdot \mathbf{r_2}$ and $\mathbf{v} \leftarrow \mathbf{m}_0\mathbf{G} + \mathbf{s} \cdot \mathbf{r}_2 + \mathbf{e}$
4c. $\mathbf{c}^* \leftarrow (\mathbf{u}, \mathbf{v})$
5. $\mathbf{b}' \leftarrow \mathcal{A}(\texttt{GUESS} : \mathbf{c}^*)$
6. $\texttt{RETURN}\ \mathbf{b}'$

As we have

$$(\mathbf{u}, \mathbf{v} - \mathbf{m}_0\mathbf{G})^\top = \begin{pmatrix} \mathbf{I}_n & \mathbf{0} & \mathcal{IM}(\mathbf{h}) \\ \mathbf{0} & \mathbf{I}_n & \mathcal{IM}(\mathbf{s}) \end{pmatrix} \cdot (\mathbf{r}_1, \mathbf{e}, \mathbf{r}_2)^\top,$$

the difference between Game $\mathbf{G}_2$ and Game $\mathbf{G}_3$ is that in the former

$$\left(\begin{pmatrix} \mathbf{I}_n & \mathbf{0} & \mathcal{IM}(\mathbf{h}) \\ \mathbf{0} & \mathbf{I}_n & \mathcal{IM}(\mathbf{s}) \end{pmatrix}, (\mathbf{u}, \mathbf{v} - \mathbf{m}_0\mathbf{G})^\top \right)$$

follows the 3-IRSD distribution (for a $[3n, n]$ ideal code), and in the latter it follows a uniform distribution (as $\boldsymbol{r}_1$ and $\mathbf{e}$ are uniformly distributed and independently chosen One-Time Pads).

Note that an adversary is not able to obtain $\boldsymbol{c}^*$ from pk any more, as depending on which game we are $\boldsymbol{c}^*$ is generated differently. The input of a game distinguisher will therefore be $(\mathsf{pk}, \boldsymbol{c}^*)$. As it must interact with the challenger as usually we suppose it has two access modes $\texttt{FIND}$ and $\texttt{GUESS}$ to process first pk and later $\boldsymbol{c}^*$.

Suppose the adversary is able to distinguish Game $\mathbf{G}_2$ and Game $\mathbf{G}_3$, with a distinguisher $\mathcal{D}_\lambda$, which takes as input $(\mathsf{pk}, \boldsymbol{c}^*)$ and outputs a guess $b' \in \{2, 3\}$ of the game we are in.

Again, we can build a distinguisher $\mathcal{D}'_{\mathcal{E},\mathcal{D}_\lambda}$ that will break the 3-$\mathsf{DIRSD}(n, \omega)$ assumption for parameters (n, ω) from $\mathsf{Setup}(1^\lambda)$ with the same advantage as the game distinguisher, when given an input $(\mathbf{H}, \mathbf{y}^\top) \in \mathbb{F}_{q^m}^{2n \times 3n} \times \mathbb{F}_{q^m}^{2n}$. In the 3-$\mathsf{DIRSD}(n, \omega)$ problem,

matrix $\mathbf{H}$ is assumed to be of the form

$$\begin{pmatrix} \mathbf{I}_n & \mathbf{0} & \mathcal{IM}(\mathbf{a}) \\ \mathbf{0} & \mathbf{I}_n & \mathcal{IM}(\mathbf{b}) \end{pmatrix}.$$

In order to use explicitly $\mathbf{a}$ and $\mathbf{b}$ we note the matrix $\mathbf{H}_{\mathbf{a},\mathbf{b}}$ instead of just $\mathbf{H}$. We will also note $\mathbf{y} = (\mathbf{y_1}, \mathbf{y_2})$.

$\mathcal{D}'_{\mathcal{E},\mathcal{D}_\lambda}((\mathbf{H}_{\mathbf{a},\mathbf{b}}, (\mathbf{y_1}, \mathbf{y_2})^\top))$
1. $\mathsf{param} \leftarrow \mathsf{Setup}(1^\lambda)$
2a. $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(\mathsf{param})$ with $\mathsf{pk} = (\mathbf{g}, \mathbf{h}, \mathbf{s} = \mathbf{x} + \mathbf{h} \cdot \mathbf{y})$ and $\mathsf{sk} = (\mathbf{x}, \mathbf{y})$
2b. $(\mathsf{pk}, \mathsf{sk}) \leftarrow ((\mathbf{g}, \mathbf{a}, \mathbf{b}), \mathbf{0})$
3. $(\mathbf{m}_0, \mathbf{m}_1) \leftarrow \mathcal{D}_\lambda(\texttt{FIND} : \mathsf{pk})$
4. $\mathbf{u} \leftarrow \mathbf{y_1}$, $\mathbf{v} \leftarrow \mathbf{m}_0\mathbf{G} + \mathbf{y_2}$ and $\boldsymbol{c}^* \leftarrow (\mathbf{u}, \mathbf{v})$
5. $\mathbf{b}' \leftarrow \mathcal{D}_\lambda(\texttt{GUESS} : \mathbf{c}^*)$
4. If $\mathbf{b}' == 2$ output $\texttt{IRSD}$
5. If $\mathbf{b}' == 3$ output $\texttt{UNIFORM}$

The distribution of pk is unchanged with respect to the games as $\mathbf{g}$ is generated by $\mathsf{KeyGen}(\mathsf{param})$ and $\mathbf{a}$ and $\mathbf{b}$ are both uniformly chosen. If $(\mathbf{H}_{\mathbf{a},\mathbf{b}}, (\mathbf{y_1}, \mathbf{y_2})^\top)$ follows the 3-$\mathsf{DIRSD}(n, \omega)$ distribution, then

$$(\mathbf{y_1}, \mathbf{y_2})^\top = \begin{pmatrix} \mathbf{I}_n & \mathbf{0} & \mathcal{IM}(\mathbf{a}) \\ \mathbf{0} & \mathbf{I}_n & \mathcal{IM}(\mathbf{b}) \end{pmatrix} \cdot (\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3})^\top$$

with $\|\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\| = \omega$. Thus, $\boldsymbol{c}^*$ follows the same distribution as in Game $\mathbf{G}_2$. If $(\mathbf{H}_{\mathbf{a},\mathbf{b}}, (\mathbf{y_1}, \mathbf{y_2})^\top)$ follows an uniform distribution, then $\boldsymbol{c}^*$ follows the same distribution as in Game $\mathbf{G}_3$. We obtain therefore the same equalities for the output probabilities of $\mathcal{D}'_{\mathcal{E},\mathcal{D}_\lambda}$ and $\mathcal{D}_\lambda$ as with the previous games and therefore the advantages of both distinguishers are equal.

Game $\mathbf{G}_4$: We now encrypt the other plaintext. We chose $\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{e}'$ uniformly at random and set $\mathbf{u} = \mathbf{r}'_1 + \mathbf{h} \cdot \mathbf{r}'_2$ and $\mathbf{v} = \mathbf{m}_1\mathbf{G} + \mathbf{s} \cdot \mathbf{r}'_2 + \mathbf{e}'$. This is the last game we describe explicitly, since, even if it is a mirror of Game $\mathbf{G}_3$, it involves a new proof.

$\mathbf{Game}^4_{\mathcal{E},\mathcal{A}}(\lambda)$
1. $\mathsf{param} \leftarrow \mathsf{Setup}(1^\lambda)$
2a. $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(\mathsf{param})$ with $\mathsf{pk} = (\mathbf{g}, \mathbf{h}, \mathbf{s} = \mathbf{x} + \mathbf{h} \cdot \mathbf{y})$ and $\mathsf{sk} = (\mathbf{x}, \mathbf{y})$
2b. $\mathbf{s} \xleftarrow{\$} \mathbb{F}_{q^m}^n$
2c. $(\mathsf{pk}, \mathsf{sk}) \leftarrow ((\mathbf{g}, \mathbf{h}, \mathbf{s}), \mathbf{0})$
3. $(\mathbf{m}_0, \mathbf{m}_1) \leftarrow \mathcal{A}(\texttt{FIND} : \mathsf{pk})$
4a. Use randomness θ to generate $\mathbf{e}' \xleftarrow{\$} \mathbb{F}_{q^m}^n$ and $\mathbf{r}' = (\mathbf{r}'_1, \mathbf{r}'_2) \xleftarrow{\$} \mathbb{F}_{q^m}^{2n}$
4b. $\mathbf{u} \leftarrow \mathbf{r}'_1 + \mathbf{h} \cdot \mathbf{r}'_2$ and $\mathbf{v} \leftarrow \mathbf{m}_1\mathbf{G} + \mathbf{s} \cdot \mathbf{r}'_2 + \mathbf{e}'$
4c. $\mathbf{c}^* \leftarrow (\mathbf{u}, \mathbf{v})$

5. $\mathbf{b}' \leftarrow \mathcal{A}(\texttt{GUESS} : \mathbf{c}^*)$
6. `RETURN` $\mathbf{b}'$

The outputs from Game $\mathbf{G}_3$ and Game $\mathbf{G}_4$ follow the exact same distribution, and therefore the two games are indistinguishable from an information-theoretic point of view. Indeed, for each tuple $(\boldsymbol{r}, \mathbf{e})$ of Game $\mathbf{G}_3$, resulting in a given $(\mathbf{u}, \mathbf{v})$, there is a one to one mapping to a couple $(\boldsymbol{r}', \mathbf{e}')$ resulting in Game $\mathbf{G}_4$ in the *same* $(\mathbf{u}, \mathbf{v})$, namely $\boldsymbol{r}' = \boldsymbol{r}$ and $\mathbf{e}' = \mathbf{e} + \mathbf{m}_0\mathbf{G} - \mathbf{m}_1\mathbf{G}$. This implies that choosing uniformly $(\boldsymbol{r}, \mathbf{e})$ in Game $\mathbf{G}_3$ and choosing uniformly $(\boldsymbol{r}', \mathbf{e}')$ in Game $\mathbf{G}_4$ leads to the same output distribution for $(\mathbf{u}, \mathbf{v})$.

Game $\mathbf{G}_5$: In this game, we now pick $\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{e}'$ with the correct weight.

Game $\mathbf{G}_6$: We now conclude by switching the public key to an honestly generated one.

We do not explicit these last two games as Game $\mathbf{G}_4$ and Game $\mathbf{G}_5$ are the equivalents of Game $\mathbf{G}_3$ and Game $\mathbf{G}_2$ except that $\mathbf{m}_1$ is used instead of $\mathbf{m}_0$. A distinguisher between these two games breaks therefore the 3-DIRSD assumption too. Similarly Game $\mathbf{G}_5$ and Game $\mathbf{G}_6$ are the equivalents of Game $\mathbf{G}_2$ and Game $\mathbf{G}_1$ and a distinguisher between these two games breaks the 2-DIRSD assumption.

We managed to build a sequence of games allowing a simulator to transform a ciphertext of a message $\mathbf{m}_0$ to a ciphertext of a message $\mathbf{m}_1$. Hence, the advantage of an adversary against the IND-CPA experiment is bounded as:

$$\mathsf{Adv}^{\mathsf{ind}}_{\mathcal{E},\mathcal{A}}(\lambda) \leq 2\left(\mathsf{Adv}^{\text{2-DIRSD}}(\lambda) + \mathsf{Adv}^{\text{3-DIRSD}}(\lambda)\right). \tag{9}$$

□

5 Known Attacks

In this section, we present the best known attacks against the DIRSD 1.1.18 problem on which RQC is based. The DIRSD problem is a decision problem. However, at the current state-of-the-art, the best attacks consist in solving an instance of the IRSD problem. There exist two types of attacks on these problems:

- the combinatorial attacks where the goal is to find the support of the error or of the codeword.
- the algebraic attacks where the opponent tries to solve an algebraic system by Groebner basis computation.

First, we deal with the combinatorial attacks then we discuss the algebraic attacks. In addition, we also provide some details regarding timing attacks against RQC.

5.1 Attack on the IRSD problem

For an $[sn, n]$ ideal code over $\mathbb{F}_{q^m}$ the best combinatorial attack to solve the IRSD problem 1.1.17 with an error of weight r is in:

$$\mathcal{O}\big(((s-1)nm)^{\omega} q^{r\left\lceil \frac{m(n+1)}{sn} \right\rceil - m}\big)$$

operations in $\mathbb{F}_q$, where ω is the exponent of the complexity of the solution of a linear system.

This attack is an improvement of a previous attack described in [12], a detailed description of the attack can be found in [3]. The general idea of the attack is to adapt the Information Set Decoding attack for the Hamming distance. For the rank metric, the attacker tries and guesses a subspace which contains the support of the error and then solves a linear system obtained from the parity-check equations to check if the choice was correct.

This attack is a generic attack against the RSD problem, there is no known improvement which exploit the ideal structure of the code.

Remark 5.1. *Since the linear system is not random, it is reasonable to take $\omega = 2$ for the choice of the parameters of RQC, even if the attack described in [3] takes $\omega = 3$.*

Let us remark that the choice of our parameter is flexible. We could take $\omega = 0$ and increase the parameters, which corresponds to only keep the exponential complexity of the attack.

5.2 Algebraic attacks

The second way to solve the equations of the system $\mathbf{He}^T = \mathbf{s}^T$ is to use the Groebner bases [18]. The advantage of these attacks is that they are independent of the size of q. They mainly depend on the number of unknowns with respect to the number of equations. However, in the case $q = 2$ the number of unknowns is generally too high for the algorithms based on Groebner bases to be more efficient than the combinatorial attacks. We have chosen our parameters such that the best attacks are combinatorial, the expected complexity of the algorithms based on Groebner bases is based on the article [6].

5.3 Quantum speed-up

For computational attacks, the quantum speed-up is easy to analyze. According to [11], a slight generalization of Grover's quantum search algorithm allows to divide by a factor 2 the exponential complexity of the attacks. Thus the complexity of the quantum computational attack is

$$\mathcal{O}\big(((s-1)nm)^{\omega} q^{\frac{1}{2}\left(r\left\lceil \frac{m(n+1)}{sn} \right\rceil - m\right)}\big)$$

for the attack on the IRSD problem.

5.4 Timing attacks

The resistance of RQC to timing attacks have been studied in [5]. This paper describes timing attacks that rely on a correlation between the weight of the error to be decoded and the running time of Gabidulin code's decoding algorithm. These attacks are of theoretical interest nevertheless are quite impracticable in real situations as they require a huge number of requests to a timing oracle. As Gabidulin code's decoding algorithm is deterministic, it can inherently be implemented in constant time in order to prevent the aforementioned attacks. Such a constant time variant of the algorithm have been provided in [5].

6 Advantages and Limitations

6.1 Advantages

The main advantages of RQC over existing code-based cryptosystems are:

- its IND-CPA reduction to a well-understood problem on coding theory: the Syndrome Decoding problem,
- its immunity against attacks aiming at recovering the hidden structure of the code being used,
- it features a null decryption failure rate ;
- it features more attractive parameters than most of the Hamming based proposals.

The null decryption failure rate allows to achieve a tight reduction for the IND-CCA2 security of the KEM-DEM version through the recent transformation of [17].

6.2 Limitations

Rank metric has very nice features, but the use of rank metric for cryptographic purposes is not very old (1991). It may seems as a limitation, but still in recent years there have been a lot of activities on understanding the inherent computational difficulty of the related problems and it seems very hard to improve on their general complexity.

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A Signed statements by the submitters

NIST requires statements about the intellectual property of the present submission. While NIST clearly mentioned they require the original paper version of these statements, the authors estimated useful to include a digital copy of these statements in this document. The paper version of these statements will be provided directly to Dustin Moody (or any other NIST member) at the first PQC Standardization Conference.

The remainder of this submission consists of statements. Below is a list of the statements included.

Statement by each submitter. Each of the authors has such a statement included.

Statement by patent owners. Carlos Aguilar Melchor and Philippe Gaborit have a patent owner statement. This patent also involves the CNRS (french national center for scientific research), represented by Éric Buffenoir. Therefore a patent statement for the CNRS is also included.

Statement by reference/optimized implementations' owners. Each of the authors has such a statement included.